Pensieve Header: Inverting R[1,1] in $\beta$-calculus and taking its square root.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-01"];
<< betaCalculus.m
```

```
{ar[1, 1], F[c[1]] ar[1, 1]} // βForm
```

$$\left\{\begin{pmatrix} 0 & h[1] \\ t[1] & 1 \end{pmatrix}, \begin{pmatrix} 0 & h[1] \\ t[1] & F[c[1]] \end{pmatrix}\right\}$$

```
ar[1, 1] ** (α ar[1, 1]) // βForm
```

$$\begin{pmatrix} 0 & h[1] \\ t[1] & 1+\alpha+\alpha\, c[1] \end{pmatrix}$$

```
Solve[1 + α + α c[1] == 0, α]
```

$$\left\{\left\{\alpha \to \frac{1}{-1-c[1]}\right\}\right\}$$

```
{R[1, 1], F[c[1]] ar[1, 1]} // βForm
```

$$\left\{\begin{pmatrix} W[1] & h[1] \\ t[1] & \frac{-1+e^{c[1]}}{c[1]} \end{pmatrix}, \begin{pmatrix} 0 & h[1] \\ t[1] & F[c[1]] \end{pmatrix}\right\}$$

```
R[1, 1] ** (F[c[1]] ar[1, 1]) // βForm
```

$$\begin{pmatrix} W[1] & h[1] \\ t[1] & \frac{-1+e^{c[1]}+e^{c[1]}\, c[1]\, F[c[1]]}{c[1]} \end{pmatrix}$$

```
R[1, 1] ** (α ar[1, 1]) // βForm
```

$$\begin{pmatrix} W[1] & h[1] \\ t[1] & \frac{-1+e^{c[1]}+e^{c[1]}\, \alpha\, c[1]}{c[1]} \end{pmatrix}$$

```
α /. First@Solve[(-1 + e^c[1] (1 + α c[1]))/c[1] == 0, α] // FullSimplify
```

$$\frac{-1+e^{-c[1]}}{c[1]}$$

```
{
  R[1, 1],
  (α ar[1, 1]) ** (α ar[1, 1])
 } // βForm
```

$$\left\{\begin{pmatrix} W[1] & h[1] \\ t[1] & \frac{-1+e^{c[1]}}{c[1]} \end{pmatrix}, \begin{pmatrix} 0 & h[1] \\ t[1] & \alpha\,(2+\alpha\, c[1]) \end{pmatrix}\right\}$$

```
α /. First@Solve[(-1 + e^c[1])/c[1] == α (2 + α c[1]), α] // FullSimplify
```

$$\frac{-1+e^{\frac{c[1]}{2}}}{c[1]}$$

```
{
  R[1, 1, -1] ** R[1, 1],
  R[1, 1, 1 / 2] ** R[1, 1, 1 / 2] ** R[1, 1, -1],
  R[1, 1, -1 / 2] ** R[1, 1, 1 / 2],
  R[1, 1, 1 / 3] ** R[1, 1, 1 / 3] ** R[1, 1, 1 / 3] // FullSimplify
 } // βForm
```

$$\left\{(W[1]),\ (W[1]),\ (W[1]),\ \begin{pmatrix} W[1] & h[1] \\ t[1] & \frac{-1+e^{c[1]}}{c[1]} \end{pmatrix}\right\}$$